SECTION - A

1. For any integral value of n; $2^n \times 5^n$ always end with zero.

So, no value of 'n' is possible.

2. Cubic polynomial whose zeros are : 3, –1 and $-\frac{1}{3}$

$$x^3-\left[3+(-1)+\left(-\frac{1}{3}\right)\right]x^2+\left[3\times(-1)+(-1)\left(-\frac{1}{3}\right)+\left(-\frac{1}{3}\right)3\right]x-3(-1)\left(-\frac{1}{3}\right)$$

$$=x^3-\frac{5}{3}x^2-\frac{11}{3}x-1 \quad \text{or} \quad 3x^3-5x^2-11x-3$$

3. For no solution: $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\Rightarrow \quad \frac{k}{6}=\frac{-5}{2}\neq\frac{2}{7}$

$\Rightarrow \quad k=-\frac{30}{2}=-15$

4. If (a, b) is equidistant from (8, 3) and (2, 7);

$\Rightarrow \quad \sqrt{(8-a)^2+(3-b)^2}=\sqrt{(2-a)^2+(7-b)^2}$

$\Rightarrow \quad 64+a^2-16a+9+b^2-6b=4+a^2-4a+49+b^2-14b$

$\Rightarrow \quad -16a-6b+4a+14b=53-73$

$\Rightarrow \quad -12a+8b=-20 \Rightarrow 2b-3a=-5$

5. Since, $\quad a_n=3n+1$

$\Rightarrow \quad a_6=3\times6+1=18+1=19$

6. 3 median = mode + 2 mean

SECTION - B

7. For real roots : D $\geq$ 0

$\Rightarrow \quad k^2-4\times16\geq0 \quad \Rightarrow \quad k^2\geq64 \quad \Rightarrow \quad k\geq8$ (for positive value)

$\therefore$ k = 8 (Least positive value)

8. **Given :** a + 2d = 4 [using a_n = a + (n – 1)d]

a + 8d = –8

$\Rightarrow$ 6d = – 12 $\Rightarrow$ d = –2

and a + (–4) = 4 $\Rightarrow$ a = 8

Now, let n^{th} term be zero

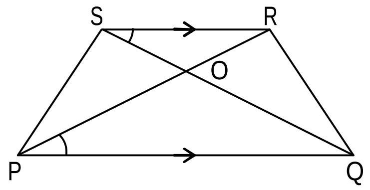
$\Rightarrow$ a_n = a + (n – 1)d

$\Rightarrow \quad 8+(n-1)(-2)=0 \quad \Rightarrow \quad n-1=\frac{-8}{-2}=4$

$\Rightarrow$ n = 5

9. **Given :** PQ|| RS

PQ = 3RS.



Since, $\Delta POQ \sim \Delta ROS$ (A.A.A similarity)

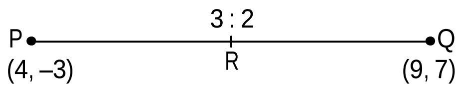
Because, $\angle POQ = \angle ROS$ (V.O.A)

$\angle OPQ = \angle ORS$ (A.I.A)

$\angle PQS = \angle QSR$ (A.I.A)

$$\Rightarrow \quad \frac{ar(\Delta POQ)}{ar(\Delta ROS)} = \frac{PQ^2}{RS^2} = \left(\frac{3RS}{RS}\right)^2 = \left(\frac{3}{1}\right)^2 = 9:1$$

10. Let the required coordinate be (x, y).



Using section formula:

$$x = \frac{(3\times 9)+(2\times 4)}{(3+2)} = \frac{27+8}{5} = \frac{35}{5} = 7$$

$$y = \frac{(3\times 7)+[2\times(-3)]}{(3+2)} = \frac{21-6}{5} = \frac{15}{5} = 3$$

So, coordinate of required point is (7, 3).

11. Since, $\tan(A+B) = \sqrt{3} = \tan 60°$

and $\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30°$

$\Rightarrow$ A + B = 60° .....(1)

$\Rightarrow$ A – B = 30° .....(2)

Solving equations (1) and (2) :

2A = 90° $\Rightarrow$ $\angle A = 45°$

From (1); $\angle B = 60° - 45° = 15°$

12. Distance to be travelled = 360 km

So, no. of days taken by first cyclist = $\frac{360}{48} = 7.5$

So, no. of days taken by second cyclist = $\frac{360}{60} = 6$

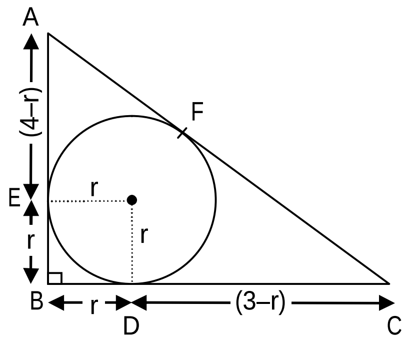
So, no. of days taken by third cyclist = $\frac{360}{72} = 5$

Since, required time = LCM (7.5, 6, 5)

= 30 days

SECTION - C

13. Let the radius of incircle be 'r' cm



Using PT : AC = $\sqrt{4^2+3^2}=5$ cm

Since, ODBE is a square.

OD = BD = OE = BE

So, DC = 3 – r, AE = 4 – r

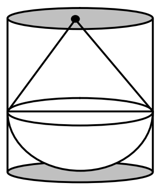
But, DC = FC and AE = AF (tangents drawn from same point)

$\Rightarrow$ AF = 4 – r and CF = 3 – r

$\Rightarrow$ (4 – r) + (3 – r) = 5 (AF + CF = AC)

$\Rightarrow$ 7 – 2r = 5 $\Rightarrow$ 2r = 2 $\Rightarrow$ r = 1 cm

14. **Given :** $h_{cone}=12$ cm; $r_{cone}=r_{cyl}=\frac{10}{2}=5\text{cm}$



Since, surface area of the toy

$= \text{CSA of cone} + \text{CSA of hemisphere}$

$= \pi r l + 2\pi r^2$

$= \pi r(l+2r)$

$= \frac{22}{7}\times 5\times(13+10)$ $\left\{l=\sqrt{h^2+r^2}=\sqrt{12^2+5^2}=\sqrt{144+25}=\sqrt{169}=13\right\}$

$= \frac{22}{7}\times 5\times 23$

$= 361.43\ \text{cm}^2$

Now, $\frac{\text{Volume of toy}}{\text{Volume of cylinder}}=\frac{\left(\frac{1}{3}\pi r^2h+\frac{2}{3}\pi r^3\right)}{\pi r^2H}$

$=\frac{\pi r^2\left(\frac{1}{3}h+\frac{2}{3}r\right)}{\pi r^2H}=\frac{\left(\frac{1}{3}\times 12\right)+\left(\frac{2}{3}\times 5\right)}{17}$ $\left(\begin{array}{l}H=h_{cone}+\text{radius of base}\\ =12+5=17\,\text{cm}\end{array}\right)$

$=\frac{4+\frac{10}{3}}{17}=\frac{22}{51}=22:51$

15. $(a-b)x+(a+b)y-(a^2-2ab-b^2)=0$

$(a+b)x+(a+b)y-(a^2+b^2)=0$

Using cross multiplication method:

$$\begin{array}{cccc} \dfrac{x}{} & \dfrac{y}{} & \dfrac{1}{} & \\ (a+b) & -(a^2-2ab-b^2) & (a-b) & (a+b) \\ (a+b) & -(a^2+b^2) & (a+b) & (a+b) \end{array}$$

$$\Rightarrow \quad \frac{x}{-(a+b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}=\frac{y}{-(a+b)(a^2-2ab-b^2)+(a^2+b^2)(a-b)}$$

$$=\frac{1}{(a-b)(a+b)-(a+b)(a+b)}$$

$$\Rightarrow \quad \frac{x}{-a^3-ab^2-a^2b-b^3+a^3-2a^2b-ab^2+a^2b-2ab^2-b^3}=\frac{1}{a^2-b^2-a^2-b^2-2ab}$$

$$\Rightarrow \quad \frac{x}{-4ab^2-2a^2b-2b^3}=\frac{1}{-2b^2-2ab}$$

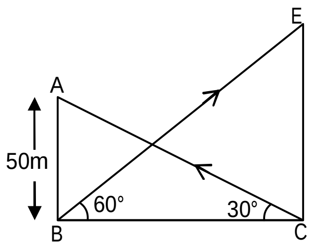
$$\Rightarrow \quad \frac{x}{-2b(a^2+b^2+2ab)}=\frac{1}{-2b(a+b)}$$

$$\Rightarrow \quad \frac{x}{-2b(a+b)^2}=\frac{1}{-2b(a+b)} \quad \Rightarrow \quad \boxed{x=(a+b)}$$

Now, $$\frac{y}{-a^3+2a^2b+ab^2-a^2b+2ab^2+b^3+a^3-a^2b+ab^2-b^3}=\frac{1}{-2b(a+b)}$$

$$\Rightarrow \quad \frac{y}{4ab^2}=\frac{1}{-2b(a+b)} \quad \Rightarrow \quad \boxed{y=-\frac{2ab}{(a+b)}}$$

16. Let AB and CE represent tower and hill respectively



Now, in ΔABC,

$$\tan 30^\circ=\frac{AB}{BC}=\frac{50}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}}=\frac{50}{BC} \qquad \Rightarrow \quad BC=50\sqrt{3}\,m$$

Now, in ΔBCE, $\tan 60^\circ=\dfrac{CE}{BC}$

$$\Rightarrow \quad \sqrt{3}=\frac{CE}{50\sqrt{3}} \quad \Rightarrow \quad CE=50\times 3=150 \text{ m}$$

$\therefore$ Height of the hill EC = 150 m

17. If two dices are thrown.

No. of possible outcomes = 6^2 = 36

(i) Sample space for the sum as prime :

{(1, 1) (1, 2) (1, 4) (1, 6) (2, 1) (2, 3) (2, 5) (3, 2) (3, 4), (4, 1) (4, 3) (5, 2) (5, 6) (6, 1) (6, 5)}

Number of favourable outcomes = 15

$\therefore$ Probability for the given event = $\frac{15}{36} = \frac{5}{12}$

(ii) Sample space for the product as perfect square

{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1), (4, 4), (5, 5), (6, 6)}

So, number of favourable outcomes = 8

$\therefore$ Probability for the given event = $\frac{8}{36} = \frac{2}{9}$

18. Let the original list price of the book be \` x.

So, number of books can be purchased = $\frac{300}{x}$

Now, if price of each book is reduced by \` 5.

So, new price of each book would be $(x - 5)$

and , number of books can be purchased now = $\frac{300}{(x-5)}$

A.T.Q. : $\frac{300}{(x-5)} - \frac{300}{x} = 5$

$\Rightarrow \quad \frac{300(x) - 300(x-5)}{x(x-5)} = 5$

$\Rightarrow \quad \frac{300(x - x + 5)}{x^2 - 5x} = \frac{5}{1}$

$\Rightarrow \quad \frac{1500}{x^2 - 5x} = \frac{5}{1} \quad \Rightarrow \quad x^2 - 5x = 300$

$\Rightarrow \quad x^2 - 5x - 300 = 0 \quad \Rightarrow \quad x^2 - 20x + 15x - 300 = 0$

$\Rightarrow \quad x(x-20) + 15(x-20) = 0$

$\Rightarrow \quad (x+15)(x-20) = 0 \quad \Rightarrow \quad x = -15 \quad \text{or} \quad 20$

$x = -15$ is impossible here.

Thus, original list price of a book is \` 20.

19. $BC = \sqrt{AC^2 + AB^2} = \sqrt{28^2 + 21^2} = \sqrt{784 + 441} = \sqrt{1225} = 35\text{ cm}$

Area of whole figure

= Area of ΔABC + Area of semicircle on AB + Area of semicircle on AC

$= \frac{1}{2} \times AB \times AC + \frac{1}{2} \times \pi \times \frac{AB}{2} \times \frac{AB}{2} + \frac{1}{2} \pi \times \frac{AC}{2} + \frac{AC}{2}$

$= \frac{1}{2} \times 21 \times 28 + \frac{1}{2} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} + \frac{1}{2} \times \frac{22}{7} \times \frac{28}{2} \times \frac{28}{2}$

$= (21 \times 14) + \frac{(33 \times 21)}{4} + (11 \times 28)$

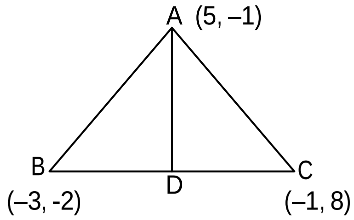
$= 294 + 173.25 + 308 = 775.25\text{ cm}^2$

Now, area of semicircle on BC = $\frac{1}{2} \pi \times \frac{BC}{2} \times \frac{BC}{2}$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{55 \times 35}{4} = \frac{1925}{4} = 481.25 \text{ cm}^2$

$\therefore$ Area of shaded region = 775.25 – 481.25 = 294 cm^2

20. Since D is the midpoint of BC

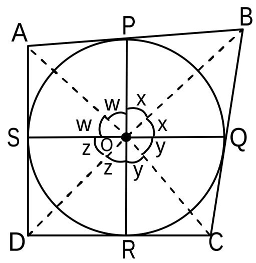


$\therefore$ coordinates of D = $\left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$ = (–2, 3)

Now, length of median AD = $\sqrt{(5+2)^2 + (-1-3)^2} = \sqrt{7^2 + (-4)^2} = \sqrt{49+16} = \sqrt{65}$ = 8.06 unit

Now, coordinates of centroid = $\left(\frac{5+(-3)+(-1)}{3}, \frac{(-1)+(-2)+8}{3}\right) = \left(\frac{1}{3}, \frac{5}{3}\right)$

21. **Given:** A circle (O, r) is inscribed in a quadrilateral ABCD.



R.T.P: $\angle$AOB + $\angle$COD = 180° and $\angle$AOD + $\angle$BOC = 180°

Construction : Join OP, OQ, OR and OS.

Proof : In ΔPOB and ΔQOB,

OP = OQ (equal radii)

BP = BQ (length of tangents drawn from ext. point B)

OB = OB (common)

$\Rightarrow$ ΔPOB $\cong$ ΔQOB (SSS congruence rule)

$\Rightarrow$ $\angle$POB = $\angle$QOB = x (CPCT)

Similarly, $\angle$QOC = $\angle$ROC = y

$\angle$ROD = $\angle$SOD = z

$\angle$SOA = $\angle$POA = w

Now, adding all angles of complete angle.

2x + 2y + 2x + 2w = 360°

$\Rightarrow$ 2(x + y + z + w) = 360°

$\Rightarrow$ x + y + z + w = 180°

$\Rightarrow$ (x + w) + (y + z) = 180°

$\Rightarrow$ $\angle$AOB + $\angle$COD = 180°

Since, it can also be written as :

(z + w) + (x + y) = 180°

$\Rightarrow$ $\angle$AOD + $\angle$BOC = 180°

22. Let the missing frequency be 'f'.

Weekly wages \` C.M. (x_i)	Number of works (f_i)	$f_i\,x_i$
41.5	31	1286.5
44.5	58	2581.0
47.5	60	2850.0
50.5	f	50.5f
53.5	27	1444.5
	$\Sigma f_i = 176 + f$	$\Sigma f_i.x_i = 8162 + 50.5F$

Since, mean (x) = $\dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\Rightarrow \quad 47.2 = \dfrac{8162 + 50.5F}{176 + f}$

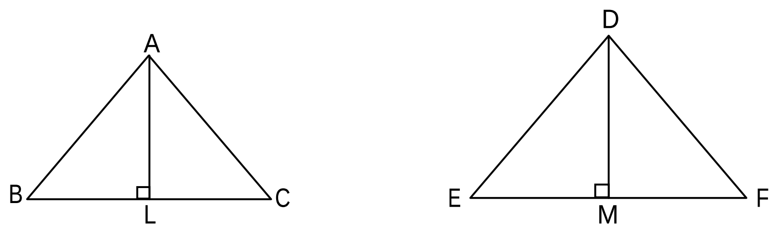
$\Rightarrow \quad 8162 + 50.5\,f = 8307.2 + 47.2\,f$

$\Rightarrow \quad 3.3\,f = 145.2$

$\Rightarrow \quad f = \dfrac{145.2}{3.3} = 44$

SECTION – D

23. **Given :** $\Delta ABC \sim \Delta DEF$, $AL \perp BC$ and $DM \perp EF$



R.T.P : $\dfrac{ar(ABC)}{ar(DEF)} = \dfrac{AL^2}{DM^2}$

Proof : In ΔABL and ΔDEM,

$\angle ABL = \angle DEM$ (C.P.S.T.)

$\angle ALB = \angle DME$ (Each 90°)

$\Rightarrow \Delta ABL \sim \Delta DEM$ (A. A. similarity criteria)

$\Rightarrow \dfrac{AB}{DE} = \dfrac{AL}{DM}$ ……(1)

Now, using $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AB^2}{DE^2}$ …..(2)

From equations (1) and (2)

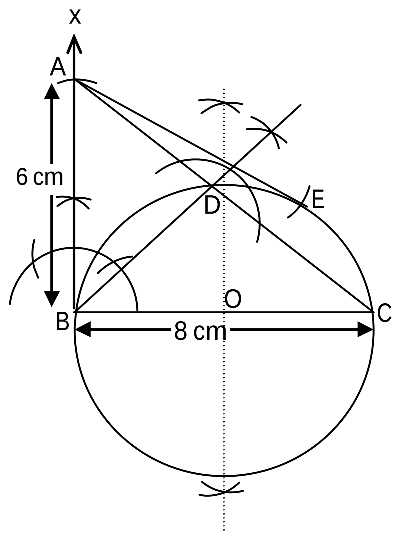
$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AL^2}{DM^2}$$

Now, if $\Delta ABC \sim \Delta PQR$

$\Rightarrow \dfrac{ar(ABC)}{ar(PQR)} = \dfrac{AB^2}{PQ^2} \Rightarrow \dfrac{81}{100} = \dfrac{AB^2}{7^2} \Rightarrow \left(\dfrac{9}{10}\right)^2 = \left(\dfrac{AB}{7}\right)^2$

$\Rightarrow \quad \frac{AB}{7} = \frac{9}{10} \quad \Rightarrow \quad AB = \frac{63}{10} = 6.3$ cm

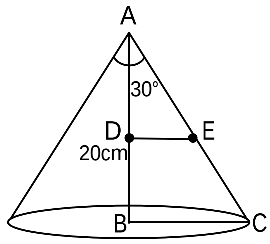
24. Steps of construction :



1. Draw line segment BC = 8cm
2. Construct an angle of 90° at B making a ray BX.
3. Cut an arc of radius 6 cm on ray BX at pt. A and Join AC
4. Draw an arc of any radius which cut AC at two points.
5. Now, draw intersecting arcs of same radius having points of intersection of previous arcs as centres.
6. Join point of intersection of intersecting arcs with B. Let it meet AC at D.
7. Draw perpendicular bisector of BC to find the centre (O) of circle which pass through B, C and D.
8. Draw a circle taking 'O' as centre and OB as radius which will pass through B, C and D.
9. Measure the length of AB with the help of compass and draw the arc of same radius = AB taking 'A' as centre which cut the circle at 'E'.
10. Join AE

    Now, AB and AE are required tangents.

25. In ΔABC,



$\tan 30° = \frac{BC}{AB}$

$\frac{1}{\sqrt{3}} = \frac{BC}{20}$

$\Rightarrow \quad BC = \frac{20}{\sqrt{3}} = R$ (let)

Now, $\Delta ADE \sim \Delta ABC$ (A.A similarity)

$\Rightarrow \quad \frac{DE}{BC} = \frac{AD}{AB} = \frac{1}{2}$

$\Rightarrow \quad DE = \frac{1}{2}BC = \frac{10}{\sqrt{3}} = r$ (let)

Since, volume of frustum = $\frac{1}{3}\pi(R^2 + r^2 + Rr)h$

$$= \frac{1}{3}\pi\left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right)10$$

$$= \frac{1}{3}\pi\left(\frac{700}{3}\right) \times 10 = \frac{7000}{9}\pi \text{ cm}^3$$

A.T.Q. : Volume of frustum = Volume of cylindrical wire

$$\frac{7000}{9}\pi = \pi \times \frac{1}{32} \times \frac{1}{32} \times \ell \qquad (\ell \text{ is the length of wire})$$

$$\Rightarrow \qquad \ell = \frac{7000 \times 32 \times 32}{9} = 796444.44 \text{ cm} = 7 \text{ km } 964 \text{ m (approx)}$$

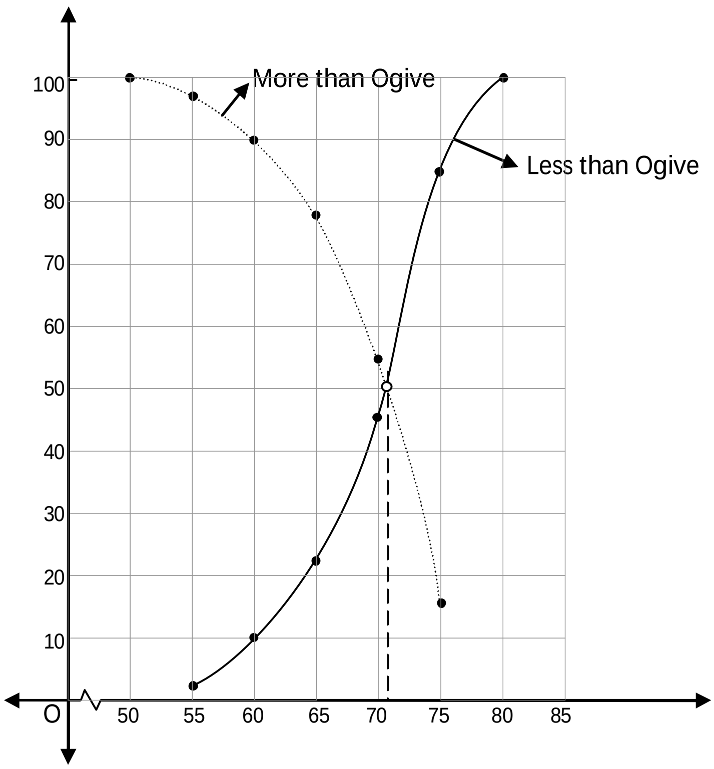
26. 'Less than' type table for the given data:

Less than	No. of forms
55	2
60	10
65	22
70	46
75	84
80	100

'More than' type table for the given data:

More than	No. of forms
50	100
55	98
60	90
65	78
70	54
75	16

Cumulative frequency curve (Ogive)



From the graph, median = 70.5

27. Solution table for $3x + y - 11 = 0$

x	3	4	5
y	2	–1	–4

Using $3x + y - 11 = 0$

(i) For $y = 2$

$3x + 2 - 11 = 0 \Rightarrow 3x = 9 \Rightarrow x = 3$

(ii) for $y = -1$

$3x - 1 - 11 = 0 \Rightarrow 3x - 12 = 0 \Rightarrow x = 4$

(iii) for $y = -4$

$3x = 15 \Rightarrow x = 5$

Solution table for $x - y - 1 = 0$

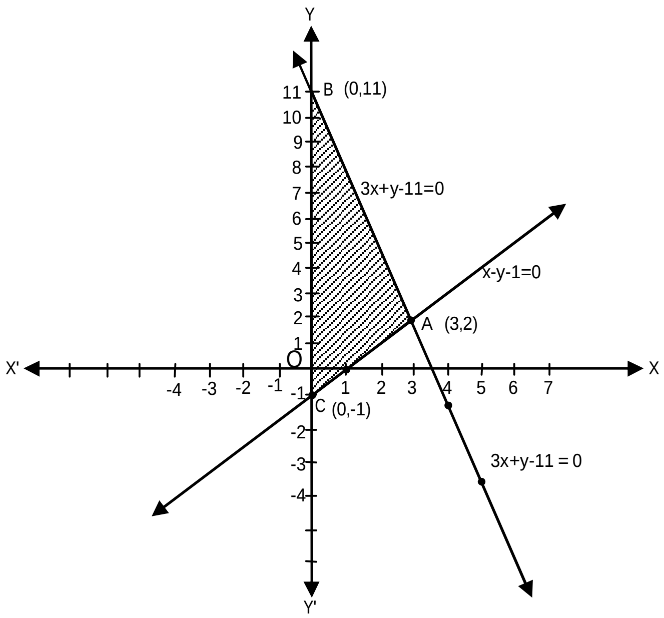
x	1	0	2
y	0	–1	1

Using $x - y - 1 = 0$

(i) for $y = 0 \Rightarrow x = 1$

(ii) for $y = 1 \Rightarrow x = 2$

(iii) for $y = -1 \Rightarrow x = 0$



Since, both lines are intersecting at (3, 2) $\Rightarrow$ $x = 3, y = 2$

Now, ABC is the required triangle.

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times 12 \times 3 = 18$ sq-units

<u>Vertices of ΔABC:</u> A(3, 2), B(0, 11), C(0, –1)

28. (i) LHS : $\frac{\cos A(1-\cos A)+\sin A(1-\sin A)+(1-\sin A)(1-\cos A)}{(1-\sin A)(1-\cos A)}$

$$= \frac{\cos A - \cos^2 A + \sin A - \sin^2 A + 1 - \cos A - \sin A + \sin A \cos A}{(1-\sin A)(1-\cos A)}$$

$$= \frac{-(\sin^2 A + \cos^2 A) + 1 + \sin A \cos A}{(1-\sin A)(1-\cos A)}$$

$$= \frac{\sin A \cos A}{(1-\sin A)(1-\cos A)} \qquad \{\text{Using } \sin^2 A + \cos^2 A = 1\}$$

= R.H.S. Hence proved.

(ii) LHS : $\sqrt{\sec^2\theta+\text{cosec}^2\theta}=\sqrt{1+\tan^2\theta+1+\cot^2\theta}$

$$=\sqrt{\tan^2\theta+\cot^2\theta+2}$$

$$=\sqrt{(\tan\theta+\cot\theta)^2}$$

$$=\tan\theta+\cot\theta$$

29. Let the incomes of Amit and Aman be `9x and `7x respectively and expenditure of Amit and Aman be `4y and `3y respectively.

A.T.Q. : $9x - 4y = 6000$ ....(1)

$7x - 3y = 6000$ ....(2)

Multiplying equation (1) by 3 and (2) by 4 and subtracting eqn. (2) from eqn. (1) :

$$\begin{array}{rcrcl} 27x & - & 12y & = & 18000 \\ \underset{-}{28x} & \underset{+}{-} & 12y & = & \underset{-}{24000} \\ \hline -x & & & = & -6000 \end{array} \Rightarrow x = 6000$$

Now from eqn. (1) :

$9 \times 6000 - 4y = 6000 \Rightarrow -4y = 6000 - 54000 \Rightarrow -4y = -48000 \Rightarrow 4y = 48000 \Rightarrow y = 12000$

So, monthly income and expenditure of Amit be `54000 and `48000 respectively and monthly income and expenditure of Aman be `42000 and `36000 respectively.

Now, amount of money donated by Amit = 2% of 54000

$=\frac{2}{100}\times 54000 =$ `1080

$\therefore$ Resulting saving of Amit = Monthly income – (Monthly expenditures + Amount of donation)

= ` 54000 – (48000 + 1080)

= ` 54000 – (49080)

= ` 4920

and amount of money donated by Aman = 2% of 42000 = `840

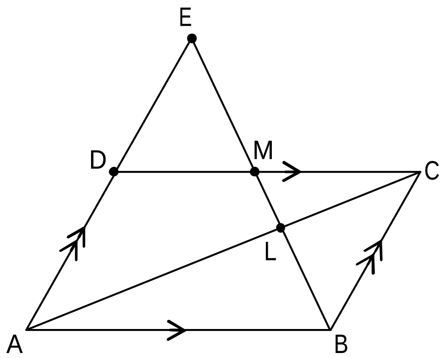
$\therefore$ Resulting saving of Aman = `42000 – (36000 + 840)

= ` 42000 – ` 36840

= ` 5160

Value : Social, Kind

30. **Given :** ABCD is a parallelogram M is the midpoint of CD, BM intersecting AC at L and meet extended AD at E.



R. T. P. : EL = 2BL

Proof : In Δs BMC and EMD, MC = MD (M is the midpoint)

$\angle BMC = \angle EMD$ (V.O.A.), $\angle BCM = \angle EDM$ (A.I.A.)

$\Rightarrow$ $\Delta BMC \cong \Delta EMD$ (ASA congruence criteria)

$\Rightarrow$ BC = DE = AD

$\Rightarrow$ AE = AD + DE = 2BC

Now, in ΔAEL and CBL, $\angle ALE = \angle CLB$ (V.O.A.)

$\angle EAL = \angle BCL$ (A.I.A.)

$\Rightarrow$ $\Delta AEL \sim \Delta CBL$ (A.A. similarity Criteria)

$\Rightarrow$ $\frac{EL}{BL} = \frac{AE}{BC} = \frac{2BC}{BC} = 2$

$\Rightarrow$ EL = 2BL Hence Proved.